Variance Risk in Aggregate Stock Returns & Time-varying Return Predictability

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Introduction

Contributions

1. New Out-of-Sample Prediction of Monthly Market Returns

Determine the risk premium of an asset by the beta representation. Use the price of variance risk (the variance risk premium, VRP), the variance risk exposure – measured using variance innovations of the market index – to compute the risk premium. **The out-of-sample R^2 is higher than 8% and is statistically significant.**

2. Time-Varying Predictive Relation

As the variance risk exposure, known as the leverage effect- varies over time, so does the predictive relation. By estimating the risk exposure at a given point of time, we can determine the predictive relation.

3. The Key is the Relation to Variance Risk - Cross-sectional Applications

The beta representation holds for every asset, and as long as an asset is exposed to market variance risk, its future returns are predictable. Assets with no correlation with market variance are not predictable.

Motivation

A. The Beta Representation

$$E_t[R_{m,t+1}] = \beta_{v,t}\lambda_{v,t} + \beta_{o,t}\lambda_{o,t}$$

- $\beta_{v,t}\lambda_{v,t}$: Risk Premium From Variance Risk (Observable)
- $\beta_{o,t}\lambda_{o,t}$: Risk Premium From Orthogonal Risk (Unobservable)

B. The Variance Risk Premium

The Variance Risk Premium (VRP)
= Option-Implied Variance (VIX^2)
– Realized Variance (RV) of S&P500 Index
$= E_t^Q[RV_{t+1}] - E_t[RV_{t+1}]$

$$VRP_t = -\lambda_{v,t}$$

C. The Leverage Effect

Market Index, Market Variance → Move in the Opposite Direction → $\beta_{v,t} < 0$

$$R_{m,t} = \beta_{v0} + \beta_v(RV_t - E_{t-1}[RV_t]) + o_t$$

D. The Predictive Relation (Bollerslev, Tauchen and Zhou, 2009)

$$R_{m,t+1} = \beta_{p0} + \beta_p VRP_t + \epsilon_{t+1}$$

1. The predictive beta, β_p in the predictive regression, may be very close to the variance risk exposure, $\beta_{v,t}$ of the contemporaneous return-variance regression in C, that comes from the leverage effect.

$$\beta_p \approx -\beta_v$$

2. The predictive power of the predictive regression in D , the R^2, strongly depends on proportion of market returns explained by variance risk.

$$R^2 \text{ of C} \approx R^2 \text{ of D}$$

Out-of-sample Prediction Methodology

The Traditional Approach of Forecast

$$R_{m,t+1} = \beta_{p0} + \beta_p VRP_t + \epsilon_{t+1}$$

$$\widehat{R_{m,t+1|t}} = \widehat{\beta_{p0}} + \widehat{\beta_p} VRP_t$$

Use long historical predictive relation to determine current relation

The Contemporaneous Beta (New) Approach

$$R_{m,t} = \beta_{v0} + \beta_v(RV_t - E_{t-1}[RV_t]) + o_t$$

$$\widehat{R_{m,t+1|t}} = \widehat{\beta_v} VRP_t + \widehat{\beta_0\lambda_o}$$

Directly use the contemporaneous variance beta as a slope of the VRP

Why is the new approach potentially better?

- The in-sample R-square for a typical predictive regression is extremely low. This means that the coefficients will be estimated with a large error.
- To get an accurate forecast we need long data, but this is impossible if the predictive relation changes over time. There are findings which suggests the predictive relations depend on economic condition.
- The R-square of the contemporaneous regression is higher than typical predictive regressions. Moreover, we can use daily data. Hence, this methodology is **adaptive to time-varying predictive relations**.

How to Estimate the Orthogonal Premium ($\beta_o\lambda_o$)?

- The VRP may be related to the price of orthogonal risk (since they are both functions of risk aversion). Let the risk premium from variance risk be determined by the variance beta and VRP, and estimate the orthogonal premium in the traditional manner, i.e. using rolling predictive regressions
- The VRP is much different from other common predictors (e.g. cay, dividend yield, or term premium) since VRP performs well in predicting short-term returns. Moreover, standard predictors perform well during recessions; the VRP works well when the leverage effect is strong.
  ➔ Thus we can use other common predictors to determine the orthogonal premium.

Out-of-sample Results

I Monthly Market Returns Predictable Out-of-Sample

		VRP_M	VRP_N
Traditional Approach (Rolling Predictive Regressions)			
7-year Rolling	OOS-R^2	0.006	−0.010
	Wald	0.021	0.037
	p-value	(0.886)	(0.848)
Expanding Window	OOS-R^2	0.039	0.025
	Wald	0.569	0.137
	p-value	(0.451)	(0.711)

$$\text{Forecast} = -\hat{\beta}_{v,T}VRP_T + \hat{\delta}_{0,T} + \hat{\delta}_T\sqrt{1-\hat{\rho}_T^2}VRP_T$$

The Contemporaneous Beta Approach			
Zero Intercept $\delta_0 = \delta_T = 0$	OOS-R^2	0.104	0.080
	Wald	8.473	4.610
	p-value	(0.004)	(0.032)
Intercept Only $\delta_T = 0$	OOS-R^2	0.096	0.075
	Wald	6.930	4.112
	p-value	(0.008)	(0.043)
VRP	OOS-R^2	0.110	0.102
	Wald	4.075	3.317
	p-value	(0.044)	(0.069)

- The traditional approach uses a rolling predictive regression to form an out-of-sample forecast.
- The beta of the contemporaneous beta approach is estimated by a regression, using daily data from a single month. The orthogonal premium is estimated by a rolling predictive regression as in the traditional approach.

Results

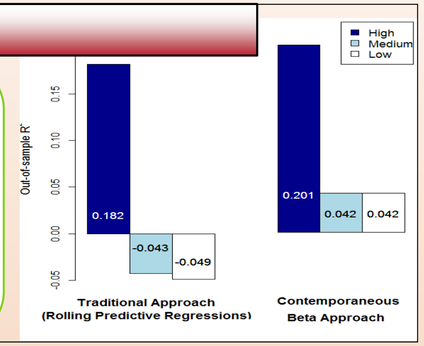
- The contemporaneous beta approach strictly outperforms the traditional approach with an out-of-sample R-squared of 8%, which is statistically significant.
- Benefits of allowing the orthogonal premium to be determined by different functional forms are small.
- A rolling predictive regression is not sufficiently reliable as the predictive relation is not stable enough.

II Conditional Predictive Performance
- Depending on the size of the leverage effect

- Divide the entire sample into high, medium, and low groups depending on the estimate of the absolute correlation between market returns and variance innovations ($|\rho(R_m, RV - E[RV])|$) – Out-of-sample classification strategy.

Results

1. Predictions are more accurate during high periods
2. The contemporaneous beta approach outperforms in all periods, but especially during low and medium periods. This is because the traditional approach overweighs information about the VRP during these periods when they are not as much useful.



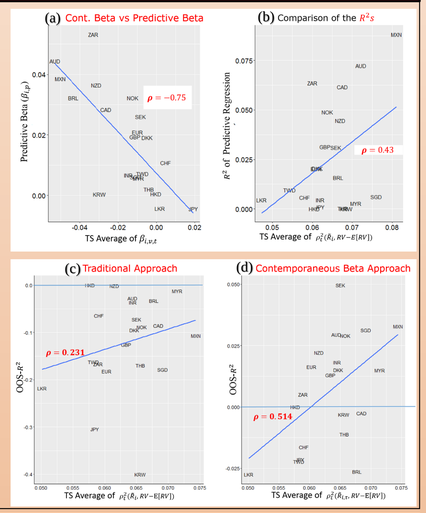
III Applications to the Cross-section of Asset Returns
- Across 21 Exchange Rates

- Any asset returns that is highly correlated with market variance may be predictable by the VRP.
- In fact, the risk premium of those assets whose returns are more exposed to variance risk may depend more on the VRP (of the equity index).
- The risk premium on assets that are more correlated to market variance predictable by a higher predictive power (R^2)

Results

1. (a) The beta of the regression
$$R_{m,t} = \beta_{v0} + \beta_v(RV_t - E_{t-1}[RV_t]) + o_t$$
closely related to the predictive beta.
2. (b) The R^2s are also closely related.
3. (c) Using the traditional approach, currency returns not predictable. Out-of-sample R^2 all negative.
4. (d) However, using the contemporaneous beta as a slope for the VRP, approximately half are predictable by a positive out-of-sample R-squared

➔ Predictability depends on variance risk exposure



Summary & Conclusion

1. Market risk premium and the VRP related in a particular way
   - Market's exposure to variance risk is close to the predictive beta
   - Monthly market returns predictable in a statistically and economically significant manner
2. Cross-sectional application to exchange rates
   - Some currency returns are predictable using the VRP, but those are substantially exposed to U.S. equity variance risk
   - The contemporaneous beta approach outperforms the UIRP relation for those currencies even out of sample

Whether, when or which returns predictable by the VRP determinable ex-ante – by variance risk exposure

References

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected Stock Returns and Variance Risk Premia, Review of Financial Studies 22, 4463-4492.